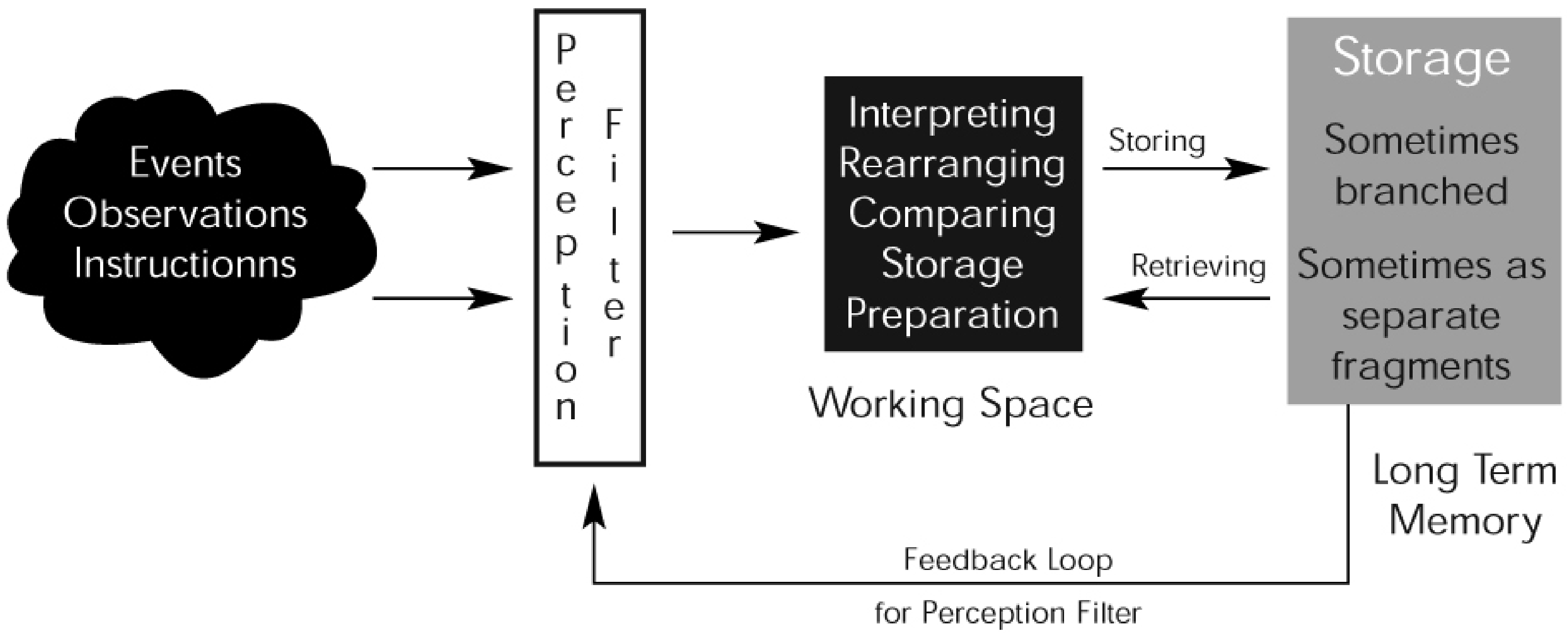
Learn How To Solve Problems


The Road
To Success

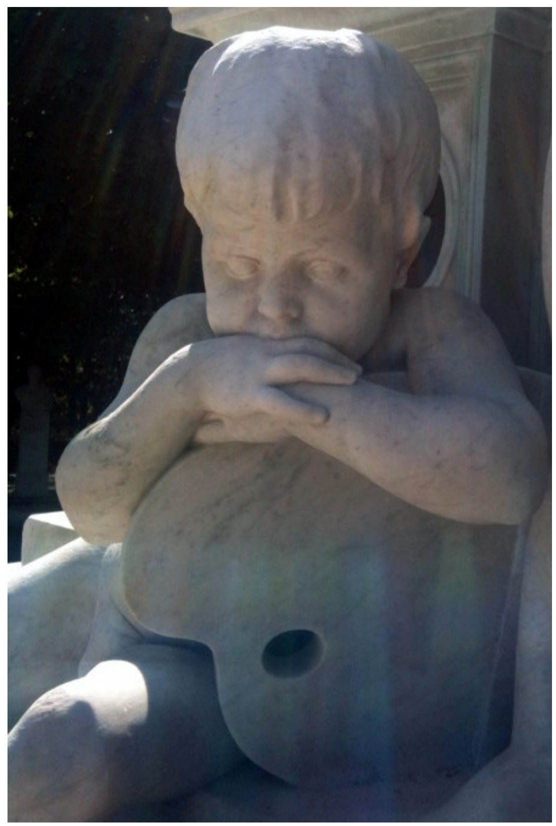
Can We Teach Problem Solving?



Information processing model (after Johnstone, A. H. *J. Chem. Educ.* **1997**, *74*, 262–268).

Background I

- Students usually act like the solution is more important than the method of finding the solution.



Background II

- Teachers seldom talk about how to find solutions to problems.

- When teachers do, students usually see a clean, even elegant solution, having little in common with the fuzzy thinking that they experience when they try to solve problems by themselves.

Problem-Solving TIPS

T – Thought Process: Think about the problem. Decide what you are being asked to find.

I – Information: Write down or highlight the key words, numbers, and facts that need to be considered.

P – Plan: Decide on a mathematical operation or strategy and set up how you will work out the problem.

S – Solution: Solve the problem by performing the strategy you chose. Don’t forget to make sure your answer makes sense by estimating and checking in your head.

Some More Tips

- Decompose problem into smaller problems
- If the problem is too hard, think of a similar problem that you can solve.
- Draw diagrams, make tables, list facts.
- Examine possible limiting cases.
- Make guesses and approach solution by iteration.

Construction of a Problem-Solving Pathway

The following material is from Dr. Joanna McCalla, St. Lawrence Campus of Champlain Regional College, Quebec, Canada.

The Problem

A 1.37 M solution of citric acid has a density of 1.10 g/mL. Determine the molality of the solution.

Objective: molality

Given: 1.37 mol $H_3C_6H_5O_7$ /L solution

1.10 g solution/cm^3 solution

Pathway:

m

What do we have to know to obtain the molality?

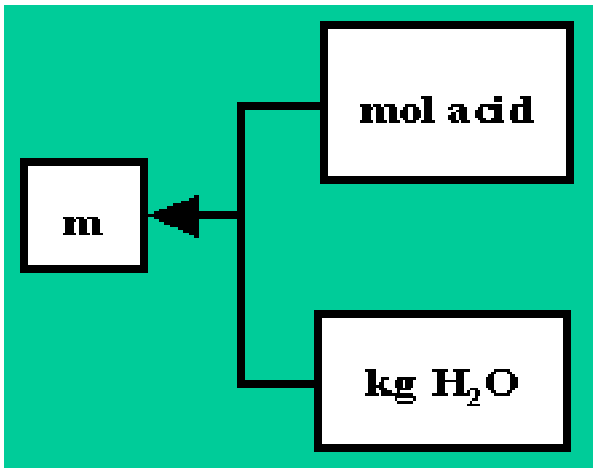
Thinking about the definition of molality...

Objective: molality

Given: 1.37 mol $H_3C_6H_5O_7$ /L solution

1.10 g solution/cm^3 solution

Pathway:



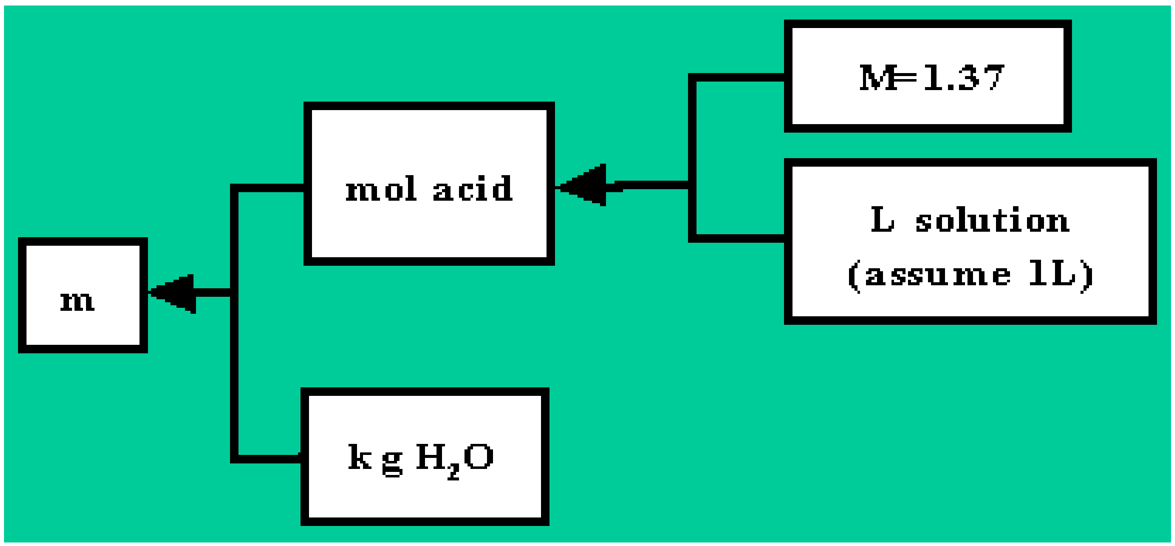
Looking at mol of acid, what do we know about moles...?

Objective: molality

Given: 1.37 mol $H_3C_6H_5O_7$ /L solution

1.10 g solution/cm^3 solution

Pathway:



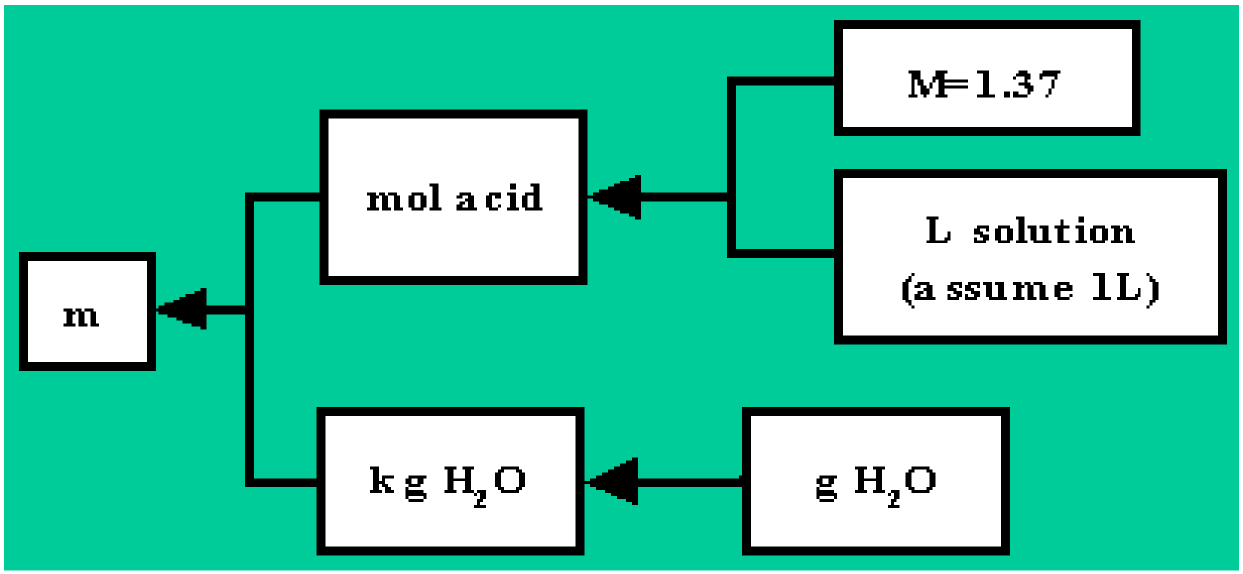
Looking at the other objective now, what we need to know to find the number of kilograms of water?

Objective: molality

Given: 1.37 mol $H_3C_6H_5O_7$ /L solution

1.10 g solution/cm^3 solution

Pathway:



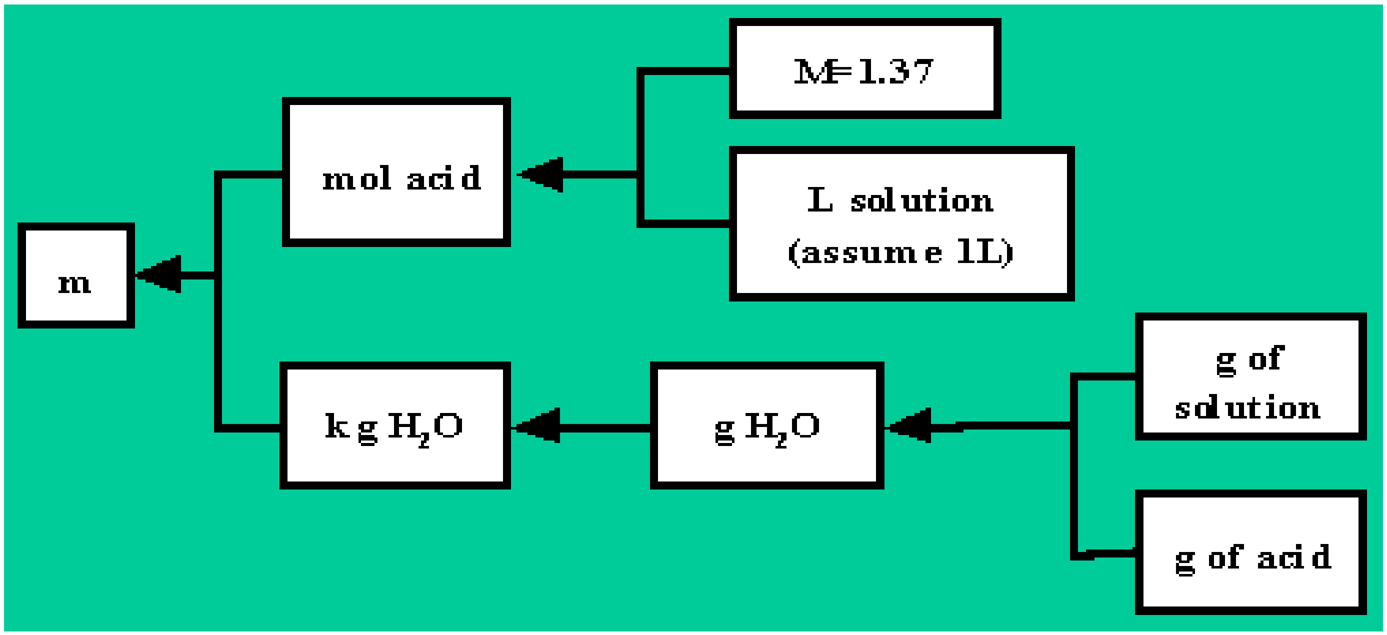
Looking now at the grams of water in the solution, let's ask the question, "What makes up the solution?"

Objective: molality

Given: 1.37 mol $H_3C_6H_5O_7$ /L solution

1.10 g solution/cm^3 solution

Pathway:



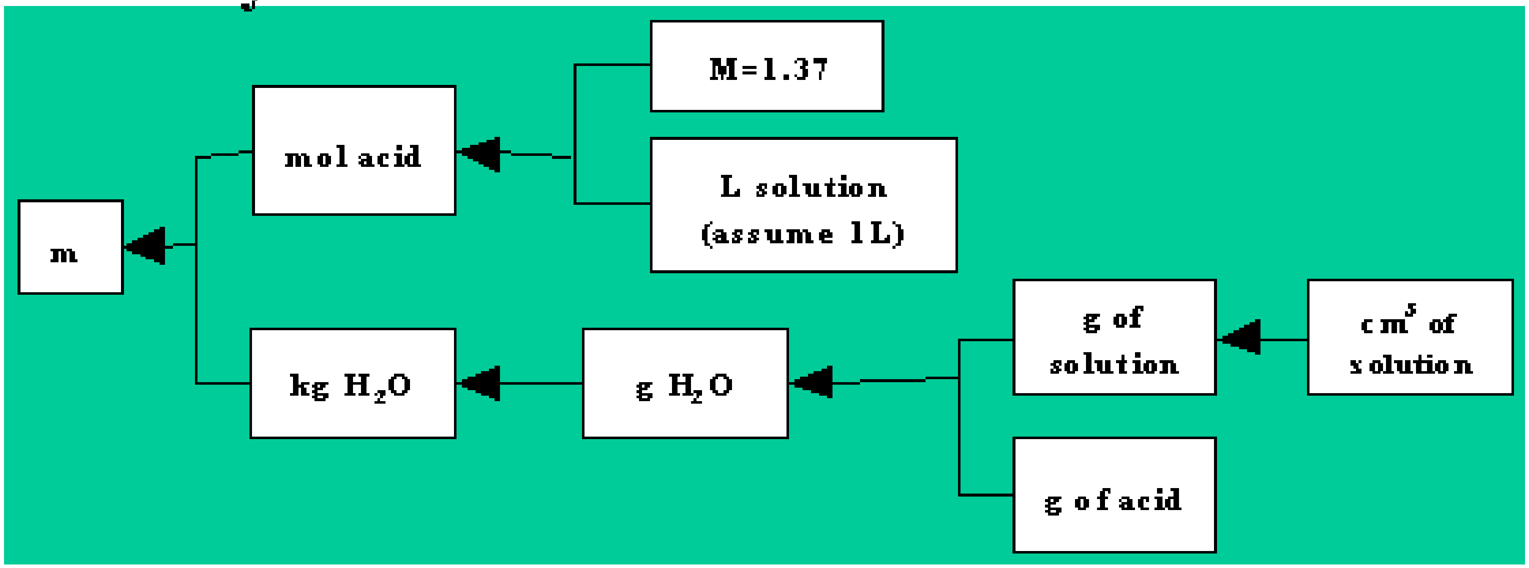
Two new objectives: what do we need to know to find the grams of solution? What has a connection with grams of solution?...

Objective: molality

Given: 1.37 mol $H_3C_6H_5O_7$ /L solution

1.10 g solution/cm^3 solution

Pathway:



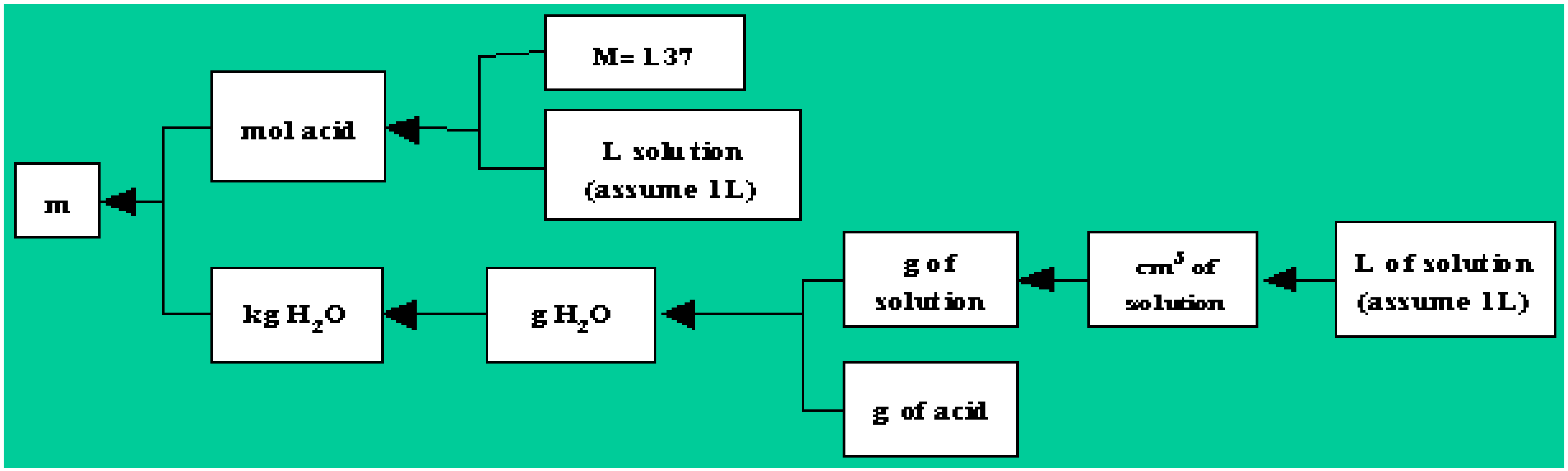
What then do we need to determine the number of cm^3 of solution? What has a connection with cm^3?...

Objective: molality

Given: 1.37 mol $H_3C_6H_5O_7$ /L solution

1.10 g solution/cm^3 solution

Pathway:



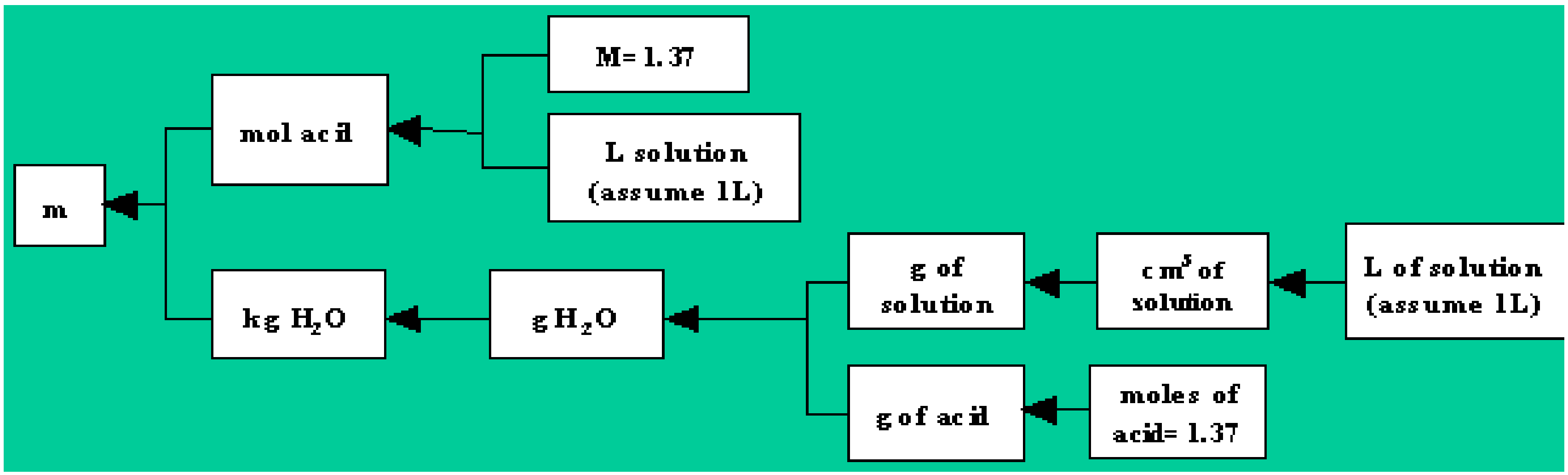
Now we only need to find the number of grams of acid.
Thinking about what we know about grams of acid...

Objective: molality

Given: 1.37 mol $H_3C_6H_5O_7$ /L solution

1.10 g solution/cm^3 solution

Pathway:



And that is it! Only the calculations remain to be done, following the pathway from right to left...

Answer: #mol acid = 1.37 mol acid (since we assumed 1L of solution)

$$\#\,\mathrm{g\ solution} = 1\,\mathrm{L} \times \frac{1000\,\mathrm{cm^3}}{\mathrm{L}} \times \frac{1.10\,\mathrm{g\ solution}}{\mathrm{cm^3}} = 1100\ \mathrm{g\ solution}$$

$$\#\,\mathrm{g\ acid} = 1.37\,\mathrm{mol\ acid} \times \frac{192\ \mathrm{g\ acid}}{\mathrm{mol\ acid}} = 263\ \mathrm{g\ acid}$$

$$\#\,\mathrm{kg\ water} = (1100 - 263)\,\mathrm{g\ water} \times \frac{1\,\mathrm{kg\ water}}{1000\,\mathrm{g\ water}} = 0.837\,\mathrm{kg\ water}$$

$$\mathrm{molality} = \frac{1.37\,\mathrm{mol\ acid}}{0.837\,\mathrm{kg\ water}} = 1.64\,\mathrm{m}$$

What Do The Experts Say?

Textbook solutions to problems provide no indication of the false starts, dead ends, illogical attempts, and wrong solutions that characterize the efforts of students when they work in problem solving.

J. D. Herron "Research in chemical education: results and directions," In M. Gardner et al., (eds), Toward a scientific practice of science education, Erlbaum, Hillsdale, NJ, p. 35 (1990).

G. M. Bodner, "Problem solving: the difference between what we do and what we tell students to do," University Chemistry Education 7, 37 (2003).

What Bodner Tells Us

An experienced teacher uses a linear, “forward-chaining” method, stringing together a logical sequence of steps and progressing smoothly from the initial information to the answer. But — a routine exercise for a teacher using a simple algorithm — becomes a challenging *novel problem* for a student who encounters this task for the first time.

More On What Bodner Tells Us

Bodner found that an “anarchistic” model describes what successful problem-solvers do when they work on novel problems in chemistry.

You try something and then you try something else if the first try fails.

Watching an instructor wade effortlessly through the task is not usually a sufficient teaching tactic. The student must stumble on his or her own personal algorithm for completing the task.

The Take-Home Message

This process of trial and error may appear disorganized or even irrational to the teacher, so that intervening to show the student the “correct” way of obtaining the answer is tempting.

While intervention may make the teacher feel good, it does not necessarily help the student!

Another Problem-Solving Strategy: Working Backwards

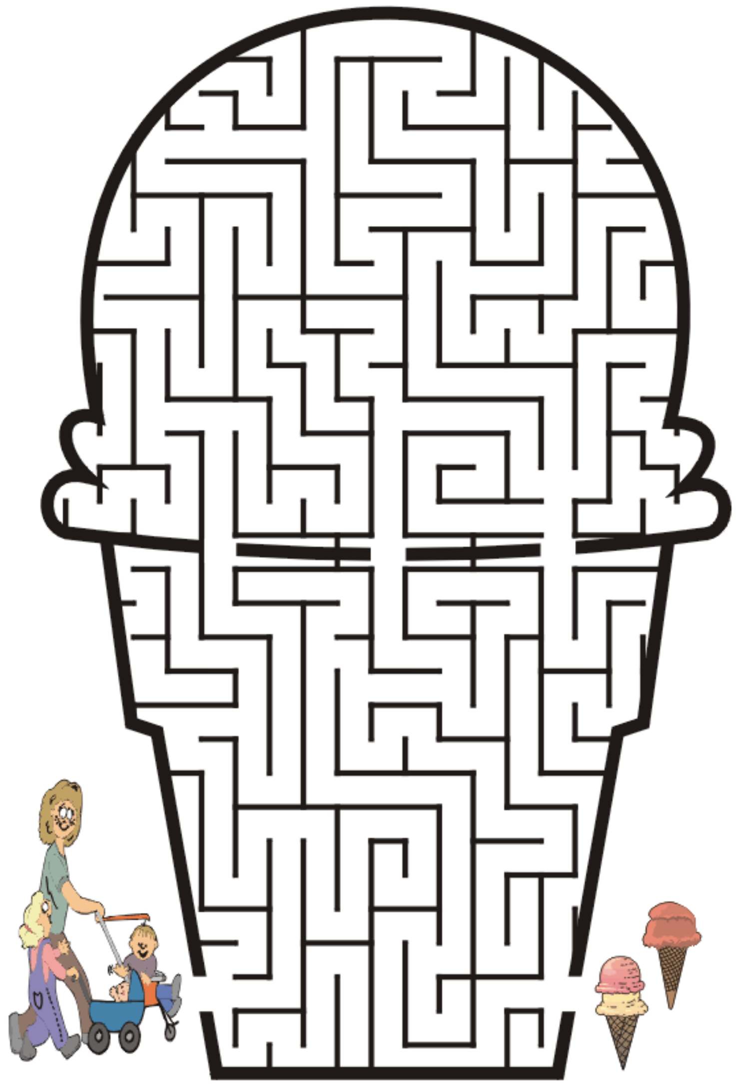
- Working Backwards has its philosophical origins in the work of the great Stanford mathematician, G. Polya, who wrote *How to Solve It* (Princeton University Press, 1957).
- Polya thought that a problem is best addressed by examining what it is that the question asks us to find, the objective, and by working backwards to the information given in the question.

Working Backwards

- We are interested in creating a problem-solving pathway that travels from the objective to the givens.

- Interestingly, the same strategy is often used in synthetic chemistry (E. J. Corey, Nobel Prize) and is called retrosynthesis.

sdrawkcaB gnikroW



Simple Example

- Problem: I have seven coins whose total value is $0.57. What coins do I have? And, how many of each coin do I have?
- Forward-going approach is to start with zero and adding all possible different combination of coins to make 57 cents.
- Backward-going approach is to start with 57 cents and think what coins are needed to make up this amount.

Nature of US Coins

Penny = one cent

Nickel = five cents

Dime = ten cents

Quarter = 25 cents

Half dollar = 50 cents

Simple Example Continued

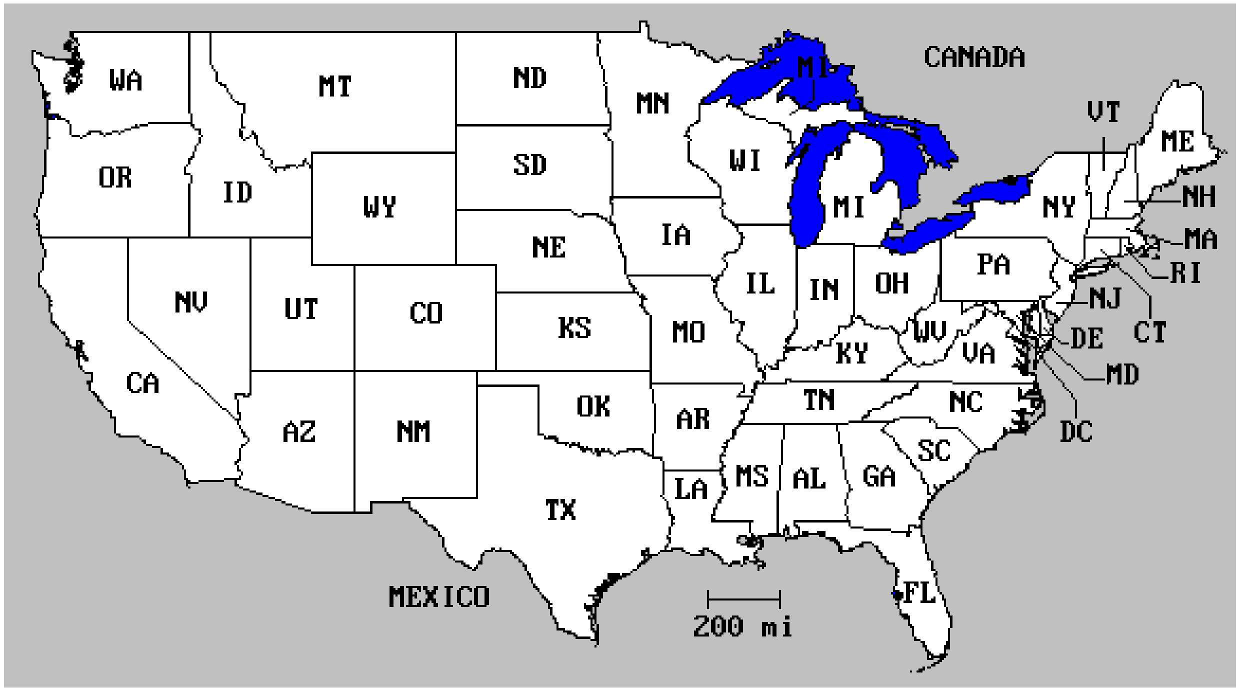
Step 1: There must be 2 pennies. (The only other option would be to use seven pennies, but that would use up all the coins prematurely.)

Step 2: Now we need to figure out how to use the 5 remaining coins to make a total of $0.55. Because 5 dimes is less than $0.55, we must use at least one quarter.

Step 3: Now we need to use 4 coins to make up the remaining $0.30. At this point, all the remaining coins must be dimes and nickels, and the only possible combination is to use 2 dimes and 2 nickels.

Another Example

You travel by car through the contiguous 48 states. The trip begins in Cedar Falls, Iowa and the rules are that you cannot enter a state more than once. What is the second and second to last state you visit in making this trip?



Herbert Simon's Model of Problem Solving:

1. **PROBLEM SPACE (all possible configurations)**

2. **PROBLEM STATE (the particular configuration)**

3. **Key to solving a problem is to choose the right OPERATORS (processes applied to change the configuration)**

4. **Problem solving is a search process: Each action takes us from one part of the problem space to another**

Means-Ends Analysis

The problem solver compares the present situation with the goal, detects a difference between them, and then searches memory for actions that are likely to reduce the difference.

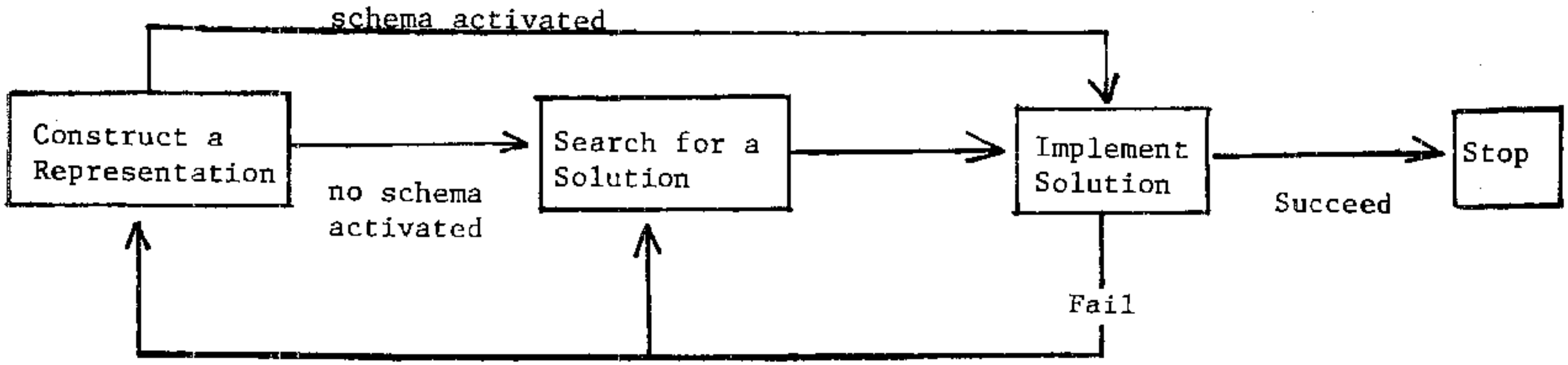
Ask yourself:

1. **What is the difference between current state and end state?**
2. **What can I do to reduce this difference?**

Make a list of means for reducing this difference.

And The Beat Goes On ...

- Mary L. Gick, Educational Psychologist 21, 99-120 (1986).
- A distinction is made between schema-driven and search-based problem-solving strategies, and expert-novice differences in the use of these strategies are discussed.



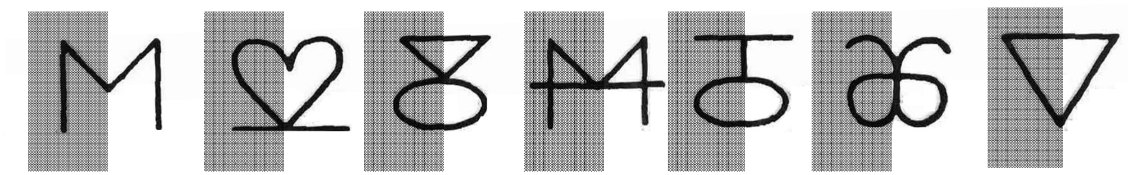
During the construction of a problem representation, certain features of the problem may activate knowledge in memory. A schema for that particular type of problem may then be activated. The schema is a cluster of knowledge related to a problem type. It contains information about the typical problem goal, constraints, and solution procedures useful for that type of problem.

Gick's Model of Problem Solving

- If schema activation should occur during the construction of a problem representation, then the solver can proceed directly to the third stage of problem solving.

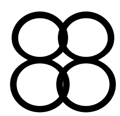
- In the absence of appropriate schema activation, the problem solver proceeds to the second step and a search strategy is invoked. Search strategies may involve the comparison of problem states to the goal state, as in means-ends analysis.

What Is The Next Member Of This Coded Series?



Look For Symmetry

The answer is

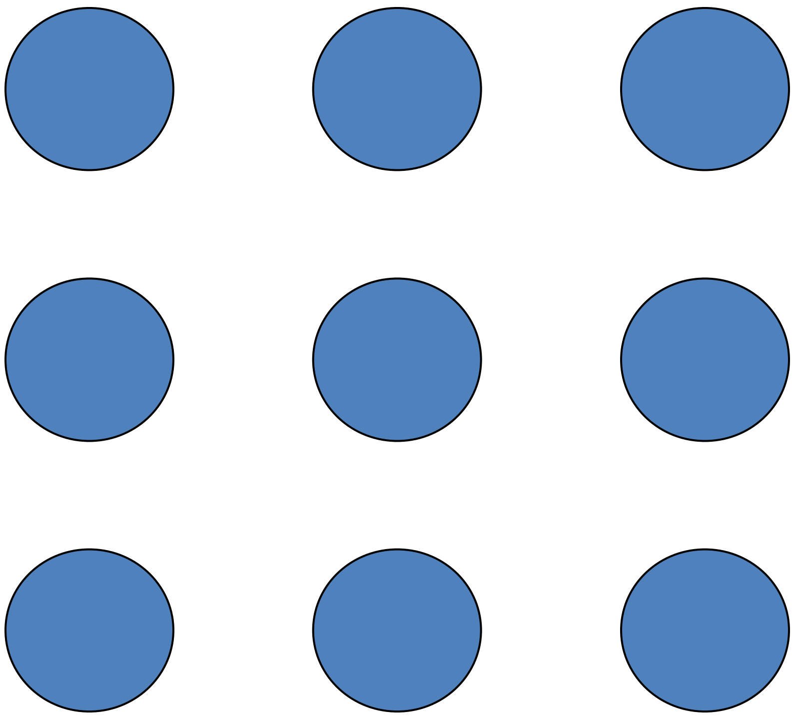


Still Another Approach

Dream About It!

Become obsessed with your problem!

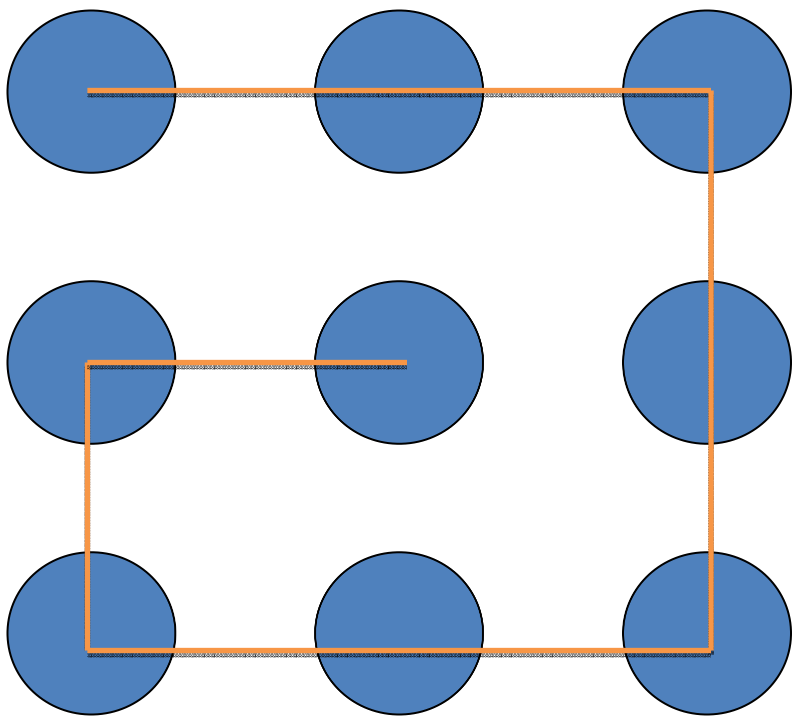
Thinking Outside the Box?



Objective: Draw four connected straight lines that pass through every circle.

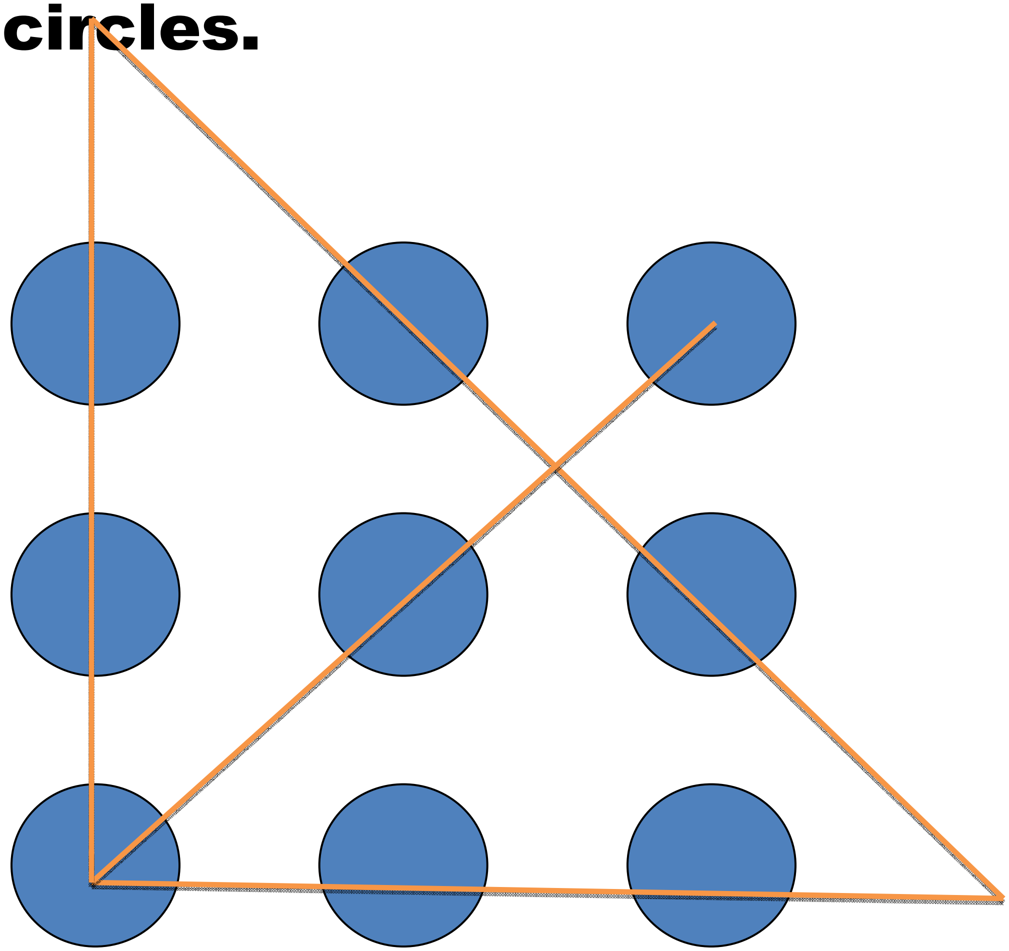
Thinking Outside the Box?

Objective: Draw four connected straight lines that pass through every circle in this 3 by 3 array of circles.



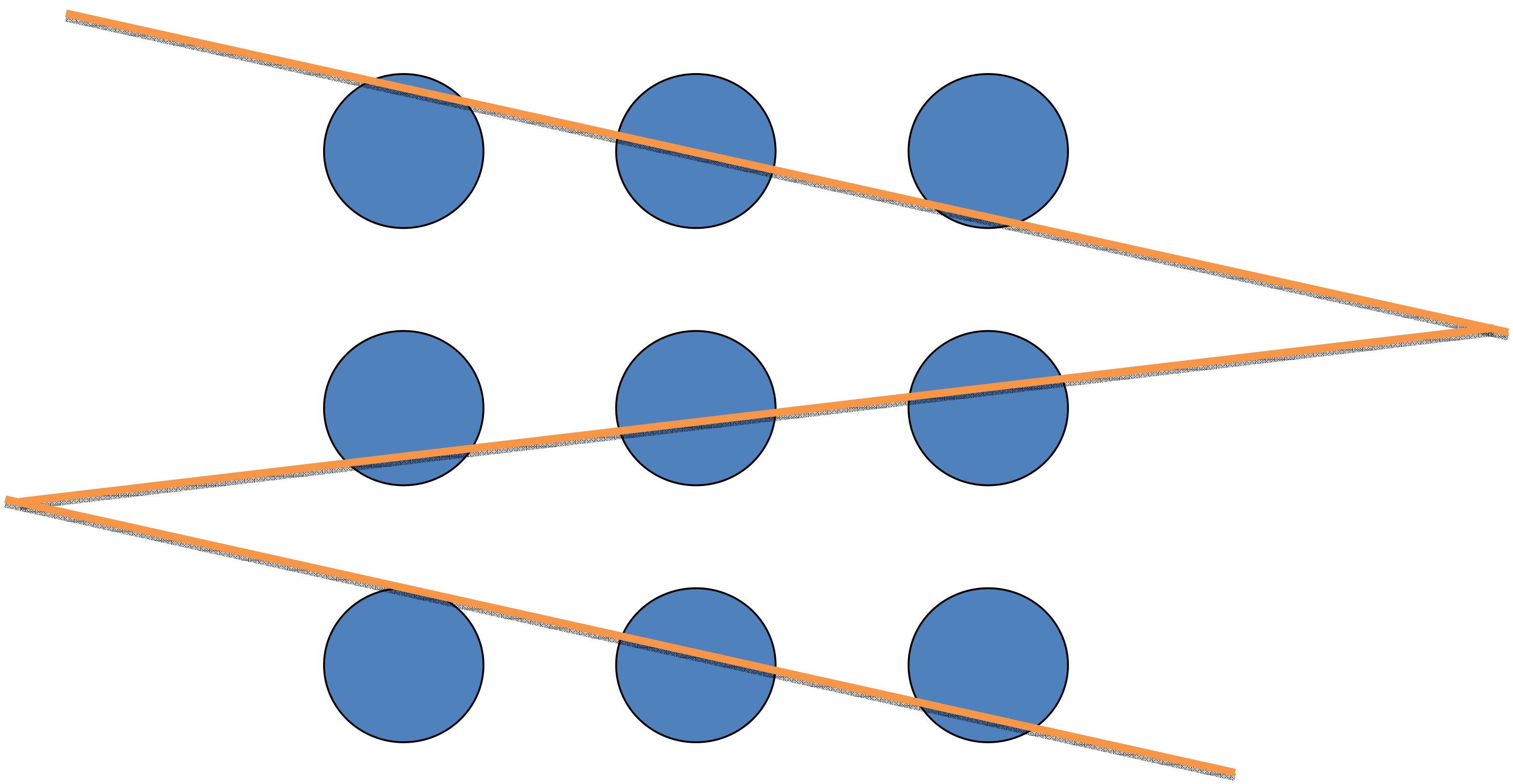
Thinking Outside the Box?

Objective: Draw four connected straight lines that pass through every circle in this 3 by 3 array of circles.



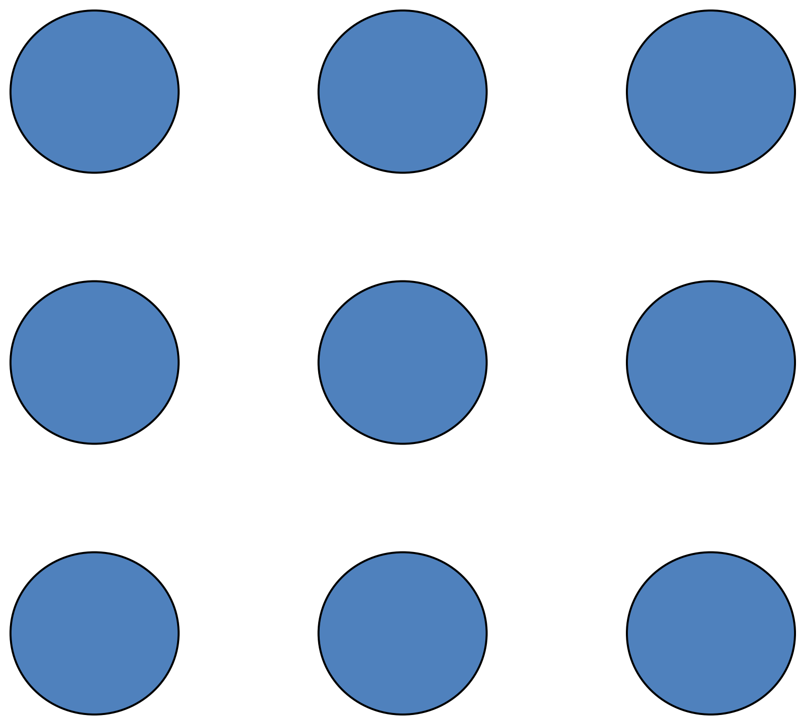
Way Outside the Box?

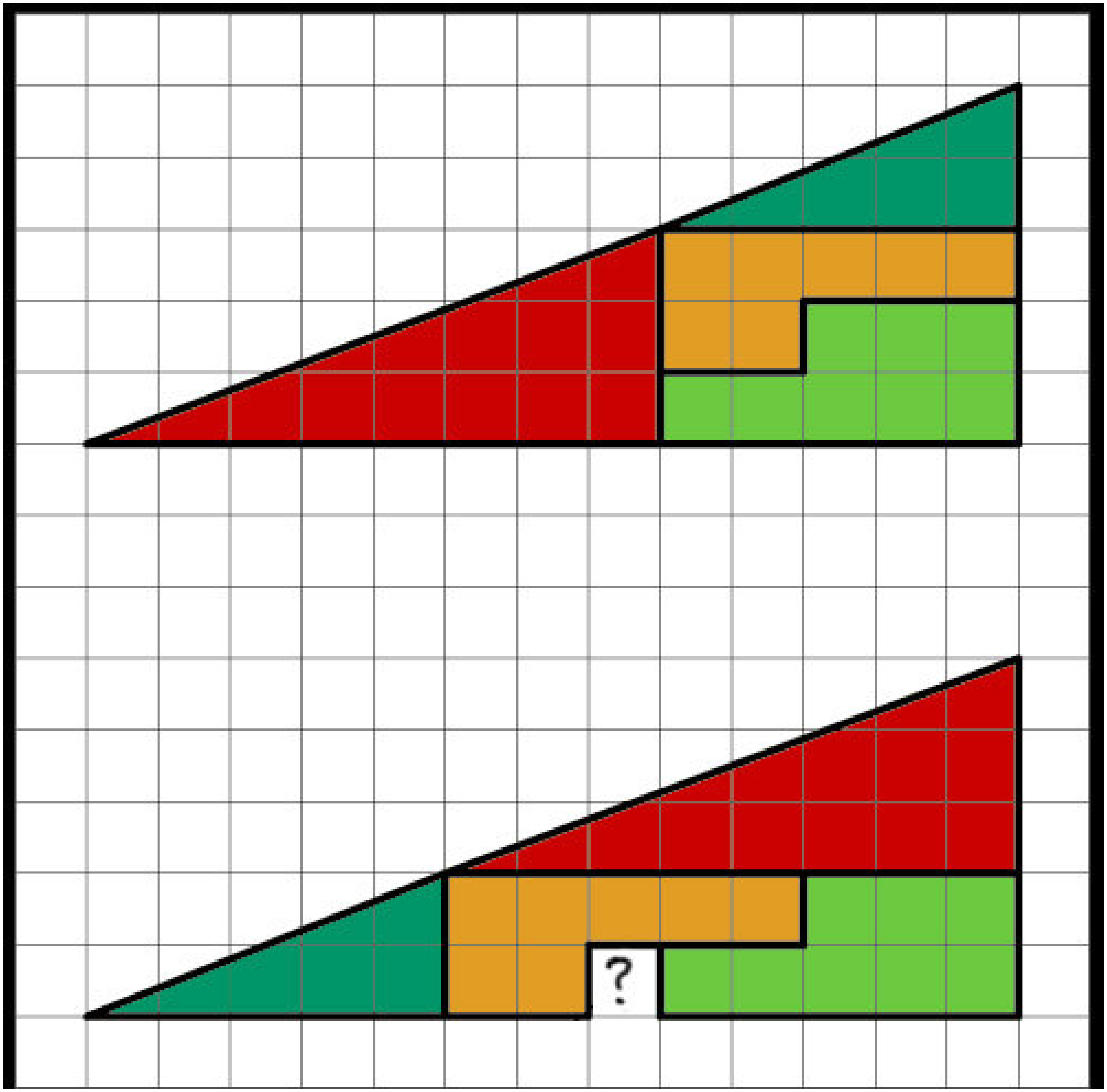
Objective: Draw three connected straight lines that pass through every circle in this 3 by 3 array of circles.

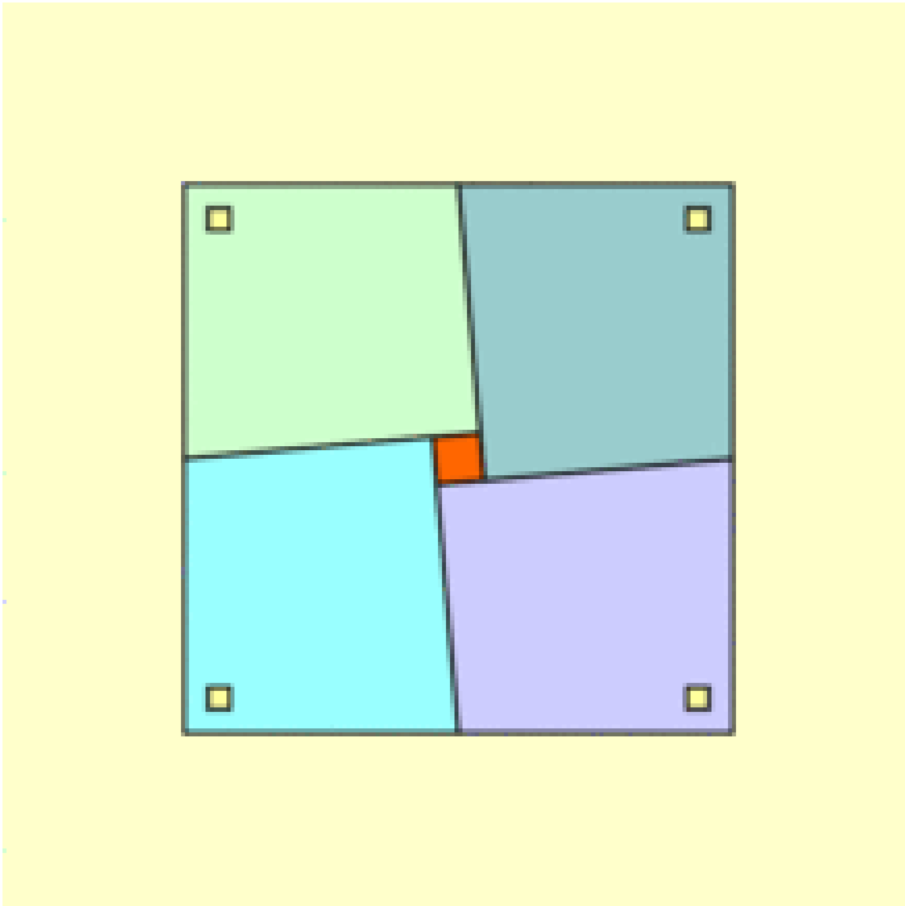


Way, Way Outside the Box?

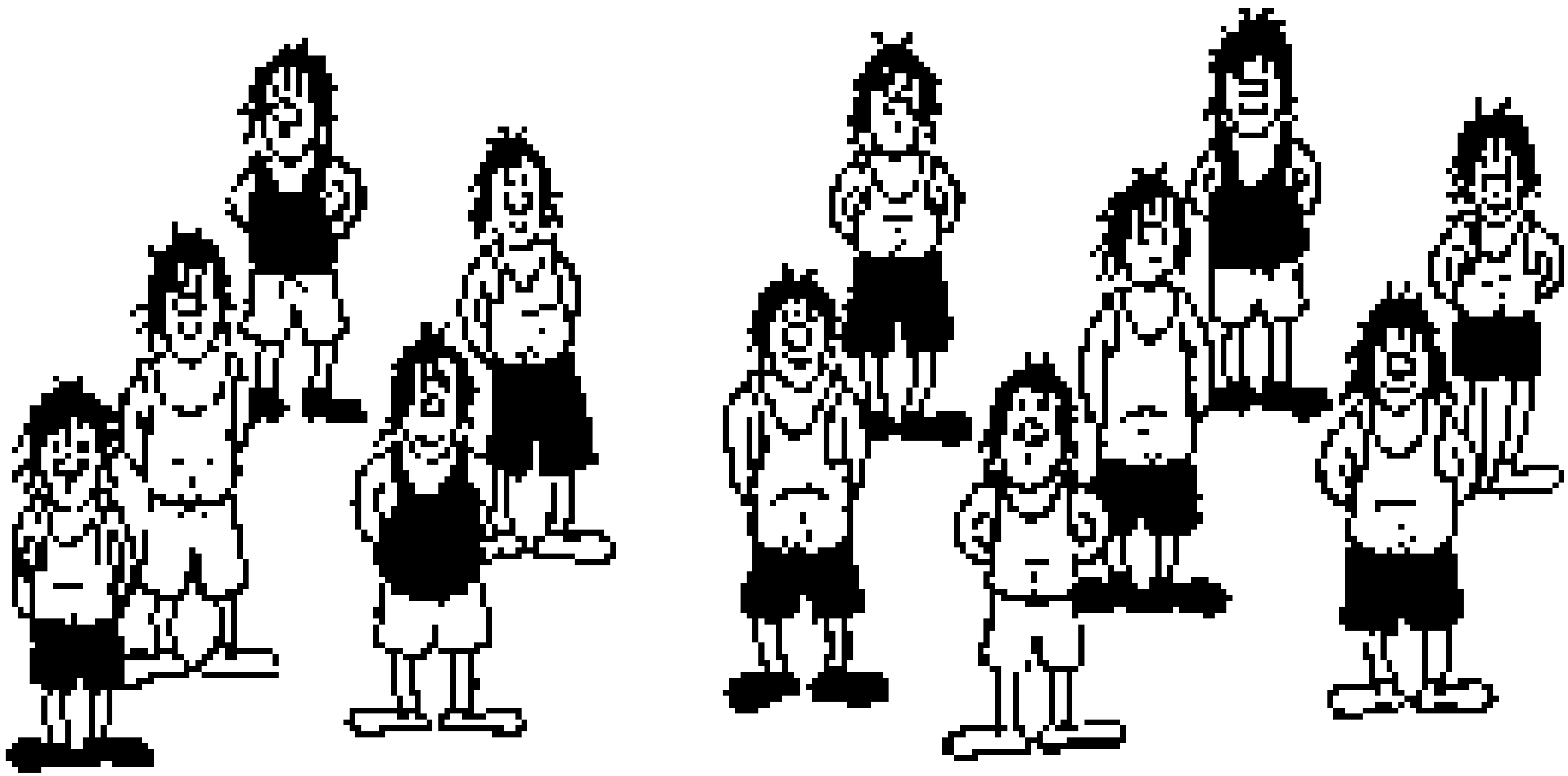
Objective: Draw one straight line that passes through every circle in this 3 by 3 array of circles.



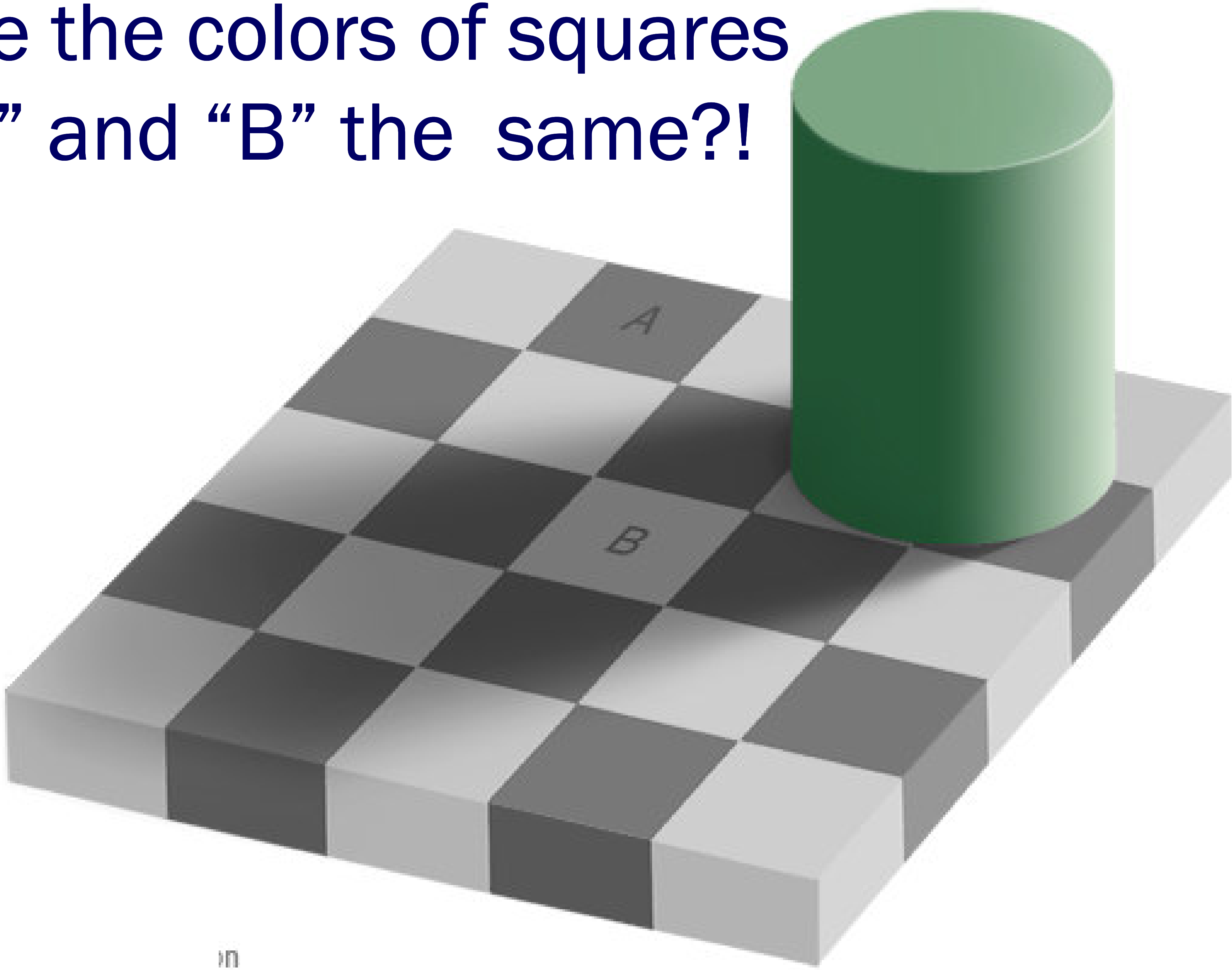


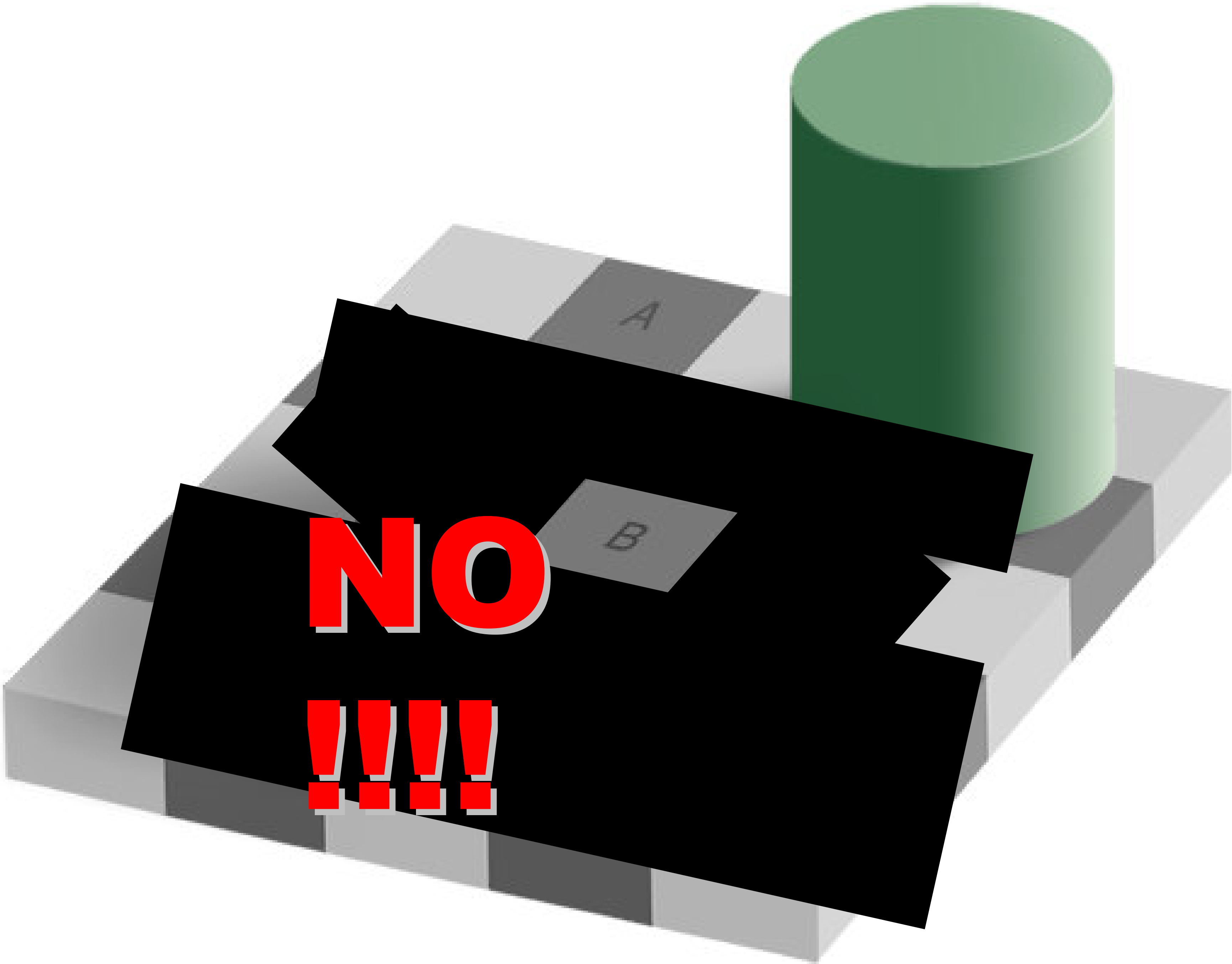


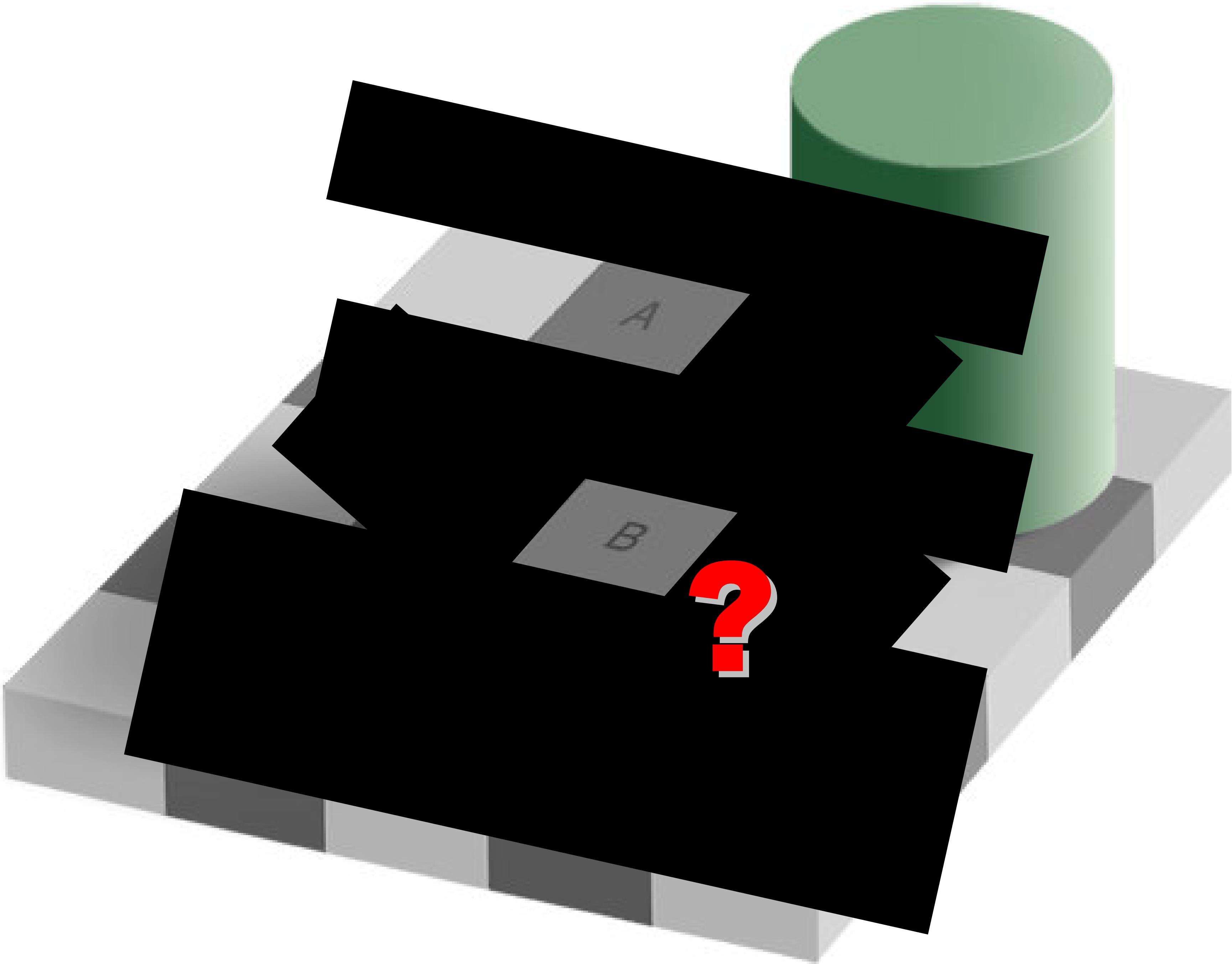
Count The Number of People



Are the colors of squares "A" and "B" the same?!



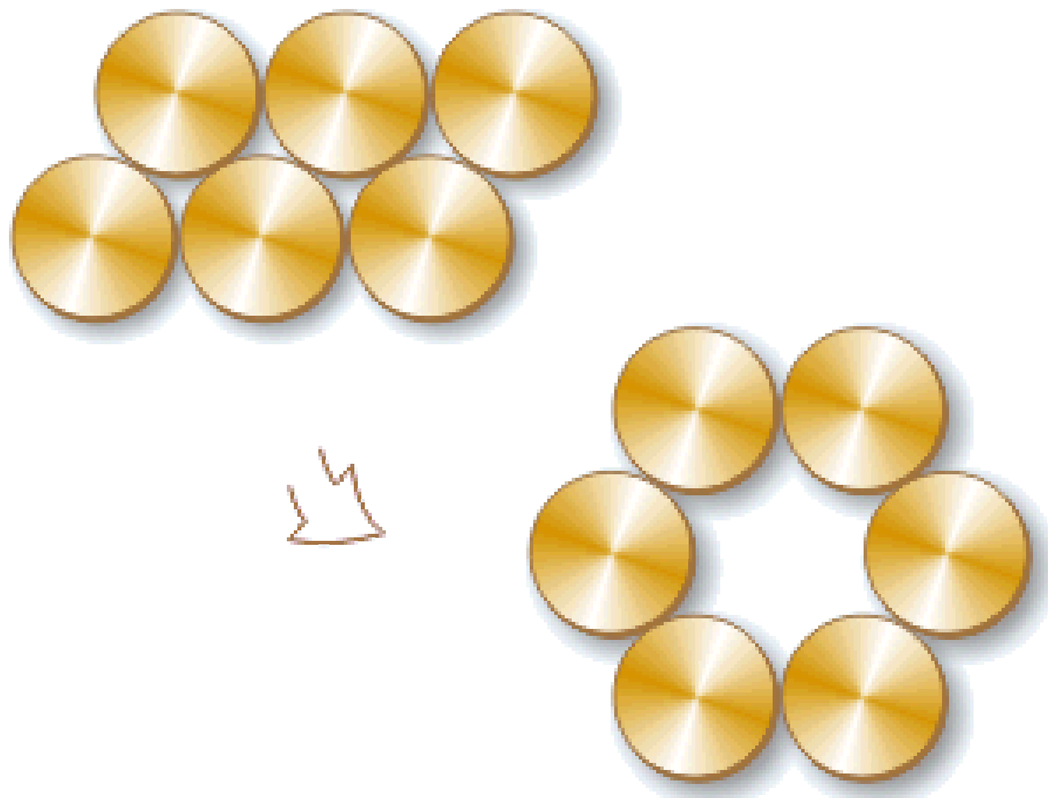

A
B
NO
!!!!


A
B
?


A
B
YES
!!!

How Many Coin Moves Are Needed?

A **move** consists of sliding one coin to a new position, where the moved coin must touch two other coins, and no other coins are allowed to move.



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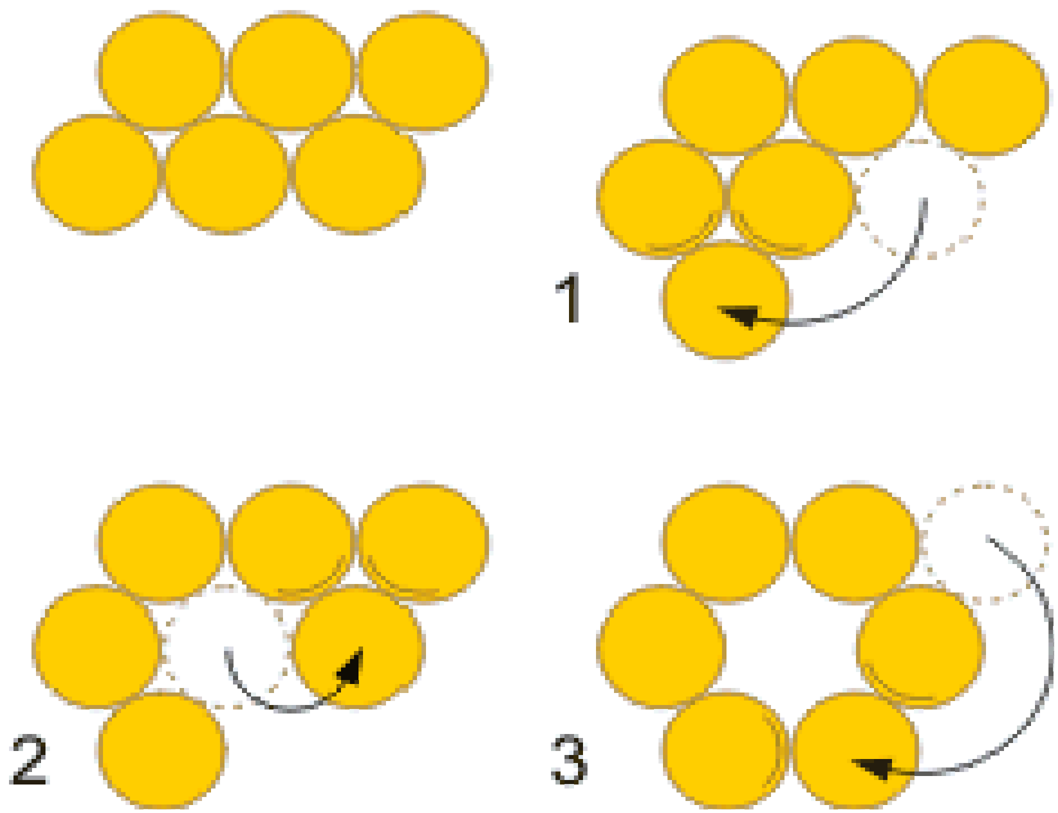


Problem Posed To Me By Liberato Cardellini

Two Italian men meet who have not seen each other in many years. While catching up on each other's news, the first discovers that the second has married and has three daughters. The first asks their ages. The second answers – "The product of their ages is 36, and their sum is equal to that house number over there" – pointing to the number under the porch of the house.

The first one replies, "I can barely see the number, but if what I see is correct, then this information is insufficient to know their ages."

"Oh, yes," replies the first, "I forgot to tell you that my youngest daughter still has blue eyes."

Tricky Reasoning

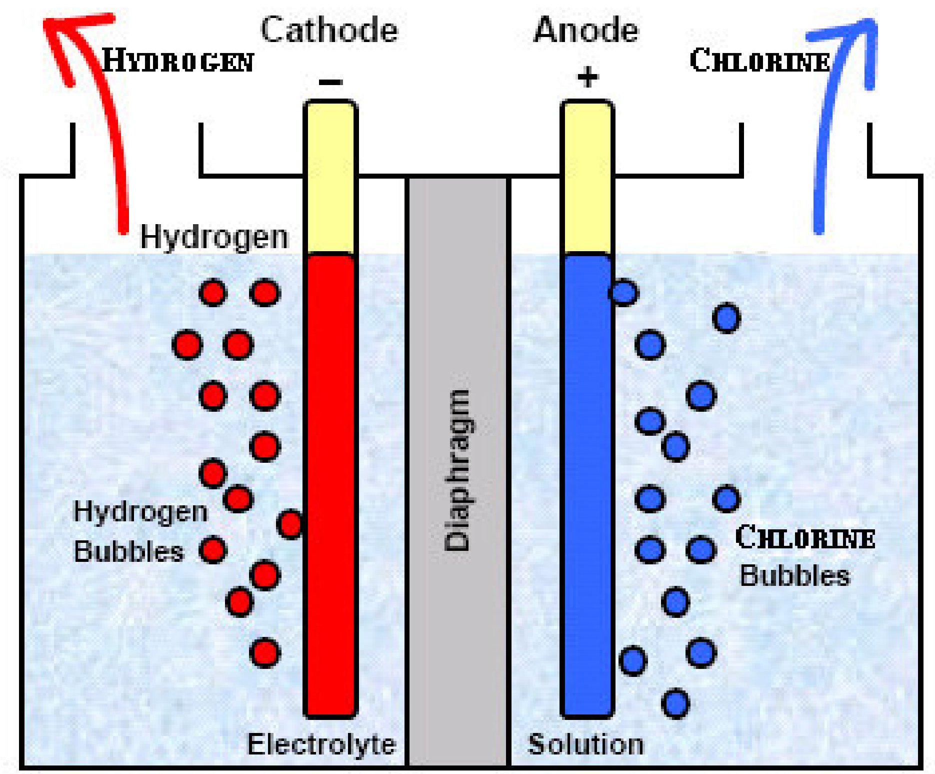
X * Y * Z = 36

X + Y + Z = "*house number there*"

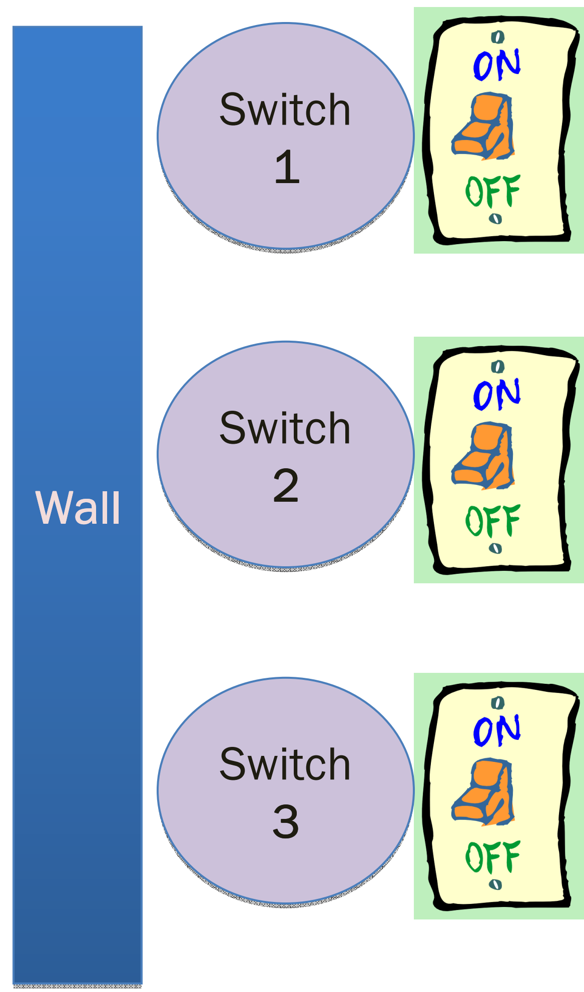
X = *has blue eyes*

1 x 1 x 36	38
1 x 2 x 18	21
1 x 3 x 12	16
1 x 4 x 9	14
1 x 6 x 6	13
2 x 2 x 9	13
2 x 3 x 6	11
3 x 3 x 4	10

Problem I Made Up



Electrolysis cell containing 1 M HCl solution is inside a sealed room having one closed door



Some Simple-Looking Problems Are <u>NOT</u> Simple

- A mixture formed by NaCl, $NaClO_3$ and $KClO_3$ contains 33.40% of oxygen and 16.00% of sodium by weight. Calculate the percentage of potassium in the mixture.

One Way of Solving This Problem

X = number of NaCl molecules
Y = number of $NaClO_3$ molecules
Z = number of $KClO_3$ molecules

S = number of Na atoms
T = number of Cl atoms
U = number of K atoms
V = number of O atoms

- K = 39, O = 16, Na = 23 and Cl = 35.5. Assume that we have 1000 grams in the mixture. Let A be Avogadro's number.

- 58.5 X/A + 106.5 Y/A + 122.5 Z/A = 1000
- 23 S/A = 160
- 16 V/A = 334
- X + Y = S
- X + Y + Z = T
- 3 Y + 3 Z = V
- Z = U

% K =

39 U / [23 S + 35.5 T + 39 U +16 V] x 100

Another Approach

Let %Na denote the percentage by weight of sodium, etc.

%Na + %O + %Cl + %K = 100

%Na = 16.00; %O = 33.40 (givens)

$\therefore$ %Cl + %K = 50.6

$\therefore$ %K = 13.56%

Mole percentage of Cl minus mole percentage of K must equal mole percentage of sodium in mixture.

%Cl/35.5 - %K/39 = %Na/23 = 16.00/23

Draw up a table of the proportion of each element in the three compounds using the relative atomic masses and relative molecular masses:

	%Na	%Cl	%O	%K
NaCl	39.32	60.68	0	0
$NaClO_3$	21.60	33.33	45.07	0
$KClO_3$	0	28.98	39.18	31.84

Assume that we start with ⅓ of each compound and calculate the amount of each element that then occurs in the mixture:

%		%Na	%Cl	%O	%K
33.33	NaCl	39.32	60.68	0	0
33.33	$NaClO_3$	21.60	33.33	45.07	0
33.33	$KClO_3$	0	28.98	39.18	31.84
	%	20.30	41.00	28.08	10.61

But we know that % of O should be 33.40% and the percentage of Na should be 16.00%.

By adjusting the percentages of NaCl and $NaClO_3$ a solution can be obtained (note the percentage of $KClO_3$ is then defined).

Starting Conditions:

After 4 iterations (the final answer):

%		%Na	%Cl	%O	%K
20.33	NaCl	39.32	60.68	0	0
37.08	$NaClO_3$	21.60	33.33	45.07	0
42.59	$KClO_3$	0	28.98	39.18	31.84
	%	16.00	37.04	33.40	13.56

In this case the solution is obtained by minimizing the parameter χ using Excel's 'Solver'.

$$\chi = (\%O - 33.40)^2 + (\%Na - 16.00)^2$$

Problem-Solving Skills Can Be Developed

- The world is a wondrous place. We advance our understand of it by posing questions and seeking answers.

- Celebrate the joy of

  aha!

  a thrilling moment of discovery and insight. It is one of the most personally rewarding aspects of teaching and learning

The Real Challenge !

- Are the givens sufficient to solve the problem?
- Are the givens as stated?
- Is the objective worth attaining? Are we asking the right question?

How to Succeed at Research

ser·en·dip·i·ty

- The faculty of making fortunate discoveries by accident.
- The fact or occurrence of such discoveries.
- An instance of making such a discovery.

SERENDIPITY

(from the Oxford English Dictionary)

f. Serendip, a former name for Sri Lanka + -ity. A word coined by Horace Walpole, who says (in a letter to Horace Mann, 28 Jan. 1754) that he had formed it upon the title of the fairy-tale `The Three Princes of Serendip', the heroes of which `were always making discoveries, by accidents and sagacity, of things they were not in quest of".

The Role of Serendipity in Scientific Research

- "In the field of observation, chance favors only the prepared mind" (Louis Pasteur)

The Power of Serendipity has been Rhapsodized by Many:

"Probably the majority of discoveries in biology and medicine have been come upon unexpectedly, or at least had an element of chance in them, especially the most important and revolutionary ones. It is scarcely possible to foresee a discovery that breaks really new ground, because it is often not in accord with current beliefs." (p. 31)

Beveridge, W. I. B. (1957). ***The art of scientific investigation.*** **New York: W. W. Norton.**

Belief that a Problem Can Be Solved

- Ice cream story
- Intermittent failure story

Importance of Confidence

Take The Zare Challenge

I am offering \$20 to the first person who solves this problem:

A mixture contains $KClO$, $MnCl_2$, and $KMnO_4$. When the percentage by weight of chlorine equals the percentage by weight of oxygen, what must be true about the percentage by weight of potassium?

For the relative atomic masses, assume: O=16, Cl=35.5, K=39, and Mn=55.

Fostering Creativity

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Stanford University

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What is Creativity?

Process of forming original ideas

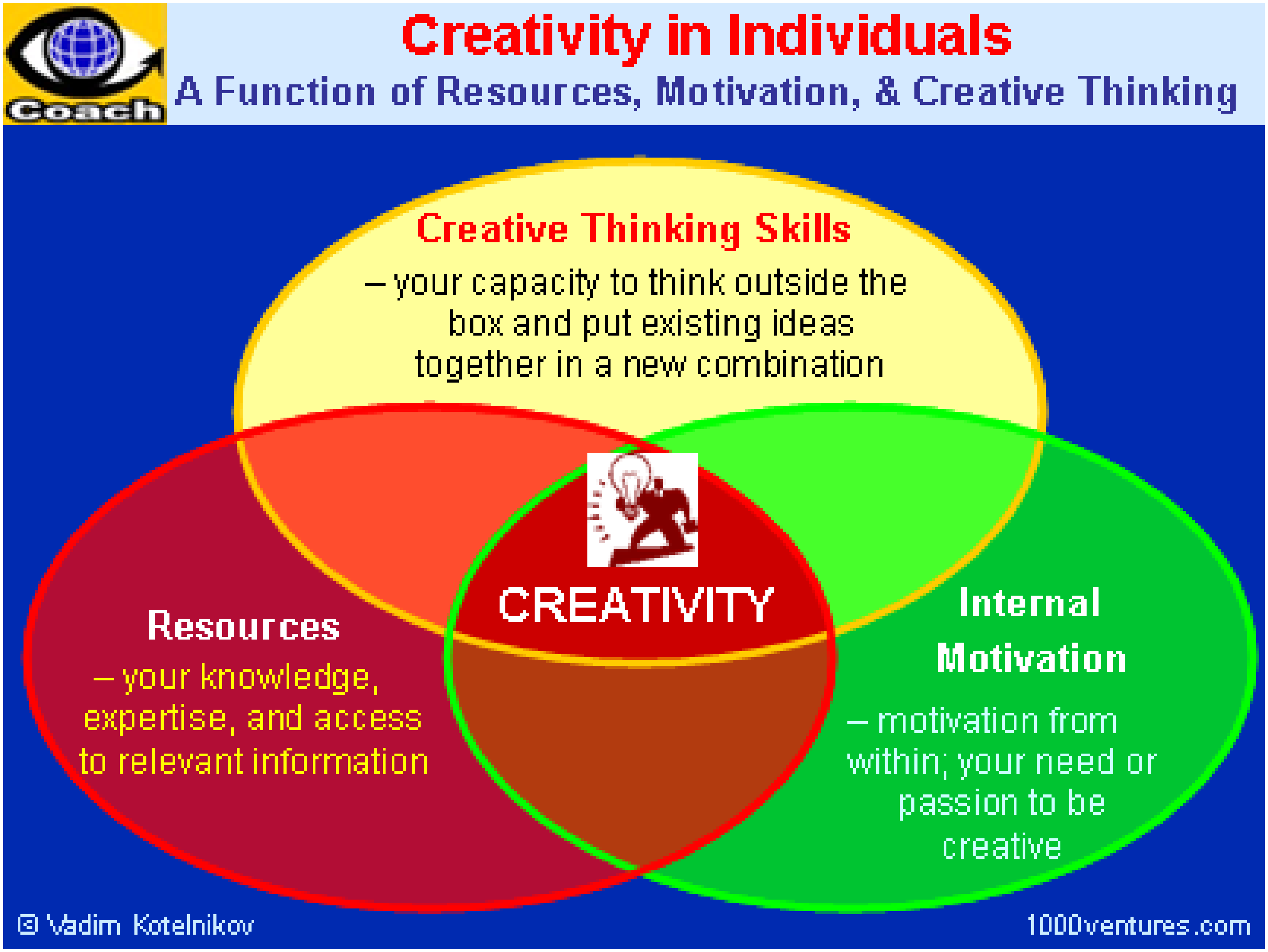
Not about talent, skill, or intelligence

Not about doing something better than others

Creativity is about thinking, exploring, discovering, imagining

Myth of the Creative Superhero

Can We Learn How To Be Creative?


Coach
Creativity in Individuals
A Function of Resources, Motivation, & Creative Thinking
Creative Thinking Skills
– your capacity to think outside the box and put existing ideas together in a new combination
CREATIVITY
Resources
– your knowledge, expertise, and access to relevant information
Internal Motivation
– motivation from within; your need or passion to be creative
© Vadim Kotelnikov
1000ventures.com

Creativity Cycle

Inertia

Seed state – waiting to grow from darkness (unknowing) to light (knowledge)

Blank page –

Scary!

Challenging!

Adaptability – an evolutionary advantage

Pearls only come from irritated oysters

Imitation

You begin art school by copying the styles of the masters you admire.

Imitation is a safe way to begin exploration

Buddha said the root of all suffering is our attachments. He also said to question everything.

Intuition

Playfulness --
a time to let go of trying to control everything

Full of fear and excitement

Become a “contented schizophrenic” believing and doubting at the same time.

Imagination

Follow your dreams

Find others to share insights and test hunches

Learn to live with chaos and ambiguity

Inspiration

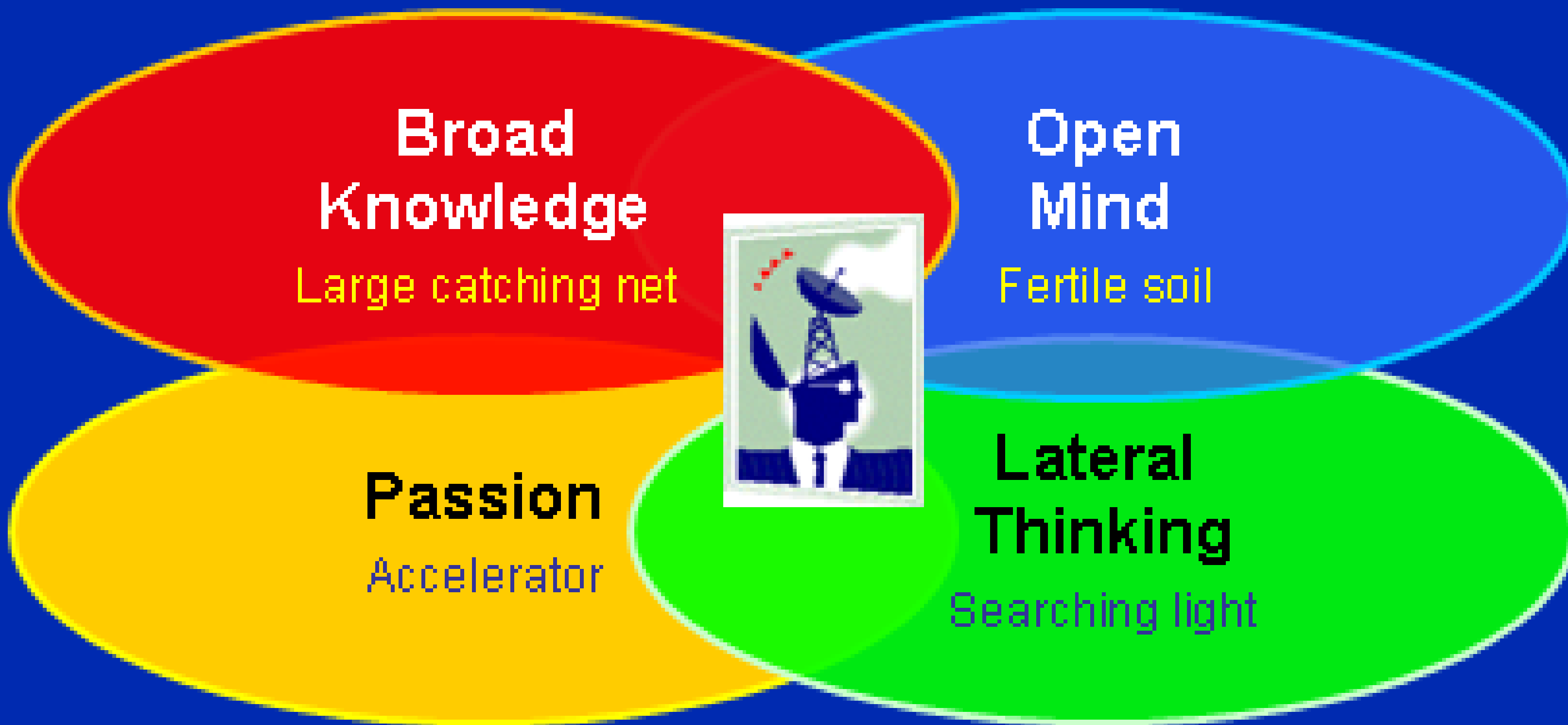
Crazy connections and interconnections

Breakthrough

Eureka! -- a moment of ecstasy!!!

Turning Accidental Discoveries Into a Habit

Four Intertwined Pillars



The Power of Active Thinking: Active thinking is about

- passionately caring about and being involved with a cause, and
- changing the problem in order to explore it

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The Power of the Question

Elements Of A Successful Career

Top Ten List:

10

Take responsibility for managing your own career. No one else will! Avoid the trap of getting caught up in the expectations of others. Polonius said it -- to thine own self be true.

9

Plan your career.

As Yogi Berra said, "If you don't know where you're going, you might end up someplace else." But life is more a stochastic process than is first imagined. One crazy thing after another keeps happening to you. Consequently, long-term planning is less useful than it might seem. When opportunity knocks, open the door. Always make plans but be flexible and be willing to reassess your plans.

8 Practice persistence.

You may or you may not need a Ph.D., but you do need dogged persistence in solving problems. Good things never come easily.

7

Don’t grow up!

Peter Pan was right. A child-like sense of wonder allows great creativity and invites discovery. You were born with this sense of wonder; don’t lose it because it is not regarded as adult behavior. Serendipity can be made to happen. Unlike lightning, once it strikes, it can be made to strike again and again.

6

Become a happy, contented schizophrenic, believing and not believing at the same time.

If you believe too easily, then you will delude yourself; if you are too critical, you will never try the outlandish. Become your own worst critic but simultaneously dare to try something different.

5

Embark on a program of continuous self-improvement.

Remember that a dull axe requires great strength to chop wood. Be wise and sharpen the blade.

4

Seek challenges.

A little known secret is that it takes about as much effort to solve a hard problem as an easy one. Don't wait for your ship to come in. Row out to meet it.

3

Go to people you trust and respect for career advice.

Recruit mentors and make friends. I cannot emphasize enough the importance of the value of critical friends, someone who will not simply tell you what you want to hear, but who will speak the plain unvarnished truth, even if it hurts. Critical friends are priceless. Of course, when you ask this of others, you must be willing to offer the same quality of friendship to others.

2 Keep your life in balance.

No job should serve as a substitute for your family or for a rich personal life. Make your work something you love. Life is short and a career is even shorter. If you don’t love your job, you better think about leaving it for some other job that you do love. Be aware that no perfect job exists. Every task has its drudgery and its frustrations. No situation is free of politics. What is important is to be able to pass through the negative so that you can dwell happily in the land of the positive. Blessed are those who achieve equanimity in this age of angst, stress, and false gods.

1

Have a dream and do something that you love.

Build sandcastles in the sky. Their foundations will follow. Select something that you love -- something that you value. Study it. Live it. Work at it. Work harder at it than you have ever worked before. Immerse yourself totally in it. In that immersion you will find happiness and contentment in a life truly well lived.